Directive: Legibly complete these exercises; turn in problems marked "TI" for *possible* grading.

1. Consider the following dictionary of predicates (over the integers).

$$
\begin{aligned}
P(x) &: x \text{ is prime} \\
E(x) &: x \text{ is even} \\
O(x) &: x \text{ is odd} \\
D(x,y) &: x \text{ divides } y \text{ (or } y \text{ is divisible by } x\text{)}
\end{aligned}
$$

   (a) Translate each of the following predicates into English.

TI i. $\forall x[P(x) \to O(x)]$

ii. $\exists x[P(x) \land E(x)]$

iii. $\forall x[O(x) \leftrightarrow (\neg E(x))]$

TI iv. $\forall x[(E(x) \to (D(x,2) \lor O(x))]$

v. $\forall x[D(1,x)]$

vi. $\forall x \exists y[x + y = 0]$

TI vii. $\exists x \forall y[x + y = 0]$

viii. $\forall x \forall y \forall z[(x * y = x * z) \to (y = z)]$

   (b) Translate each of the following statements using the dictionary above.

TI i. Every integer is either even or odd.

TI ii. If 3 divides integer x, then x is odd.

TI iii. If x is prime and x is greater than two, then x is odd.

iv. An integer is divisible by 6 if and only if it is divisible by both 2 and 3.

TI v. For every integer n, if n is strictly greater than two, then for all positive integers x, y, z the equation $x^n + y^n = z^n$ does not hold.

2. Translate the predicate $C((a_n)_{n\in\mathbb{N}}, L)$ about sequences of real numbers into symbolic logic: "For every $\epsilon > 0$ there exists a natural number N such that for all $n \geq N$ we have $|a_n - L| < \epsilon$." What does this predicate define?

TI 3. Let $A = \{1, 3, 5\}$, $B = \{-1, 1, 2, 4\}$, and $C = \{2, 4, 6\}$. Compute each of the following.

   (a) $A \cap B$

   (b) $A \cup C$

   (c) $B \setminus C$

   (d) $B \times A$

   (e) $\mathbb{P}(A)$